SADLER MATHEMATICS SPECIALIST UNIT 2

WORKED SOLUTIONS

Chapter 9 Trigonometrical identities and equations

Exercise 9A

Question 1

$$\begin{aligned}\text{LHS} &= 2\cos^2\theta + 3\\ &= 2(1-\sin^2\theta) + 3\\ &= 2 - 2\sin^2\theta + 3\\ &= 5 - 2\sin^2\theta\\ &= \text{RHS}\end{aligned}$$

Question 2

$$\begin{aligned}\text{LHS} &= \sin\theta - \cos^2\theta\\ &= \sin\theta - (1-\sin^2\theta)\\ &= \sin\theta + 1 + \sin^2\theta\\ &= \sin\theta(1+\sin\theta) + 1\\ &= \text{RHS}\end{aligned}$$

Question 3

$$\begin{aligned}\text{LHS} &= (\sin\theta + \cos\theta)^2\\ &= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta\\ &= \sin\theta + \cos^2\theta + 2\sin\theta\cos\theta\\ &= 1 + 2\sin\theta\cos\theta\\ &= \text{RHS}\end{aligned}$$

Question 4

$$\begin{aligned}\text{LHS} &= 1-2\sin\theta\cos\theta\\ &= \sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta\\ &= \left(\sin\theta-\cos\theta\right)^2\\ &= \text{RHS}\end{aligned}$$

Question 5

$$\begin{aligned}\text{LHS} &= \sin^4\theta-\cos^4\theta\\ &= \left(\sin^2\theta-\cos^2\theta\right)\left(\sin^2\theta+\cos^2\theta\right)\\ &= \left(\sin^2\theta-\cos^2\theta\right)\times 1\\ &= \sin^2\theta-(1-\sin^2\theta)\\ &= \sin^2\theta-1+\sin^2\theta\\ &= 2\sin^2\theta-1\\ &= \text{RHS}\end{aligned}$$

Question 6

$$\begin{aligned}\text{LHS} &= \sin^2\theta(\sin^2\theta-1)\\ &= \left(1-\cos^2\theta\right)\left(1-\cos^2\theta-1\right)\\ &= \left(1-\cos^2\theta\right)(-\cos^2\theta)\\ &= -\cos^2\theta+\cos^4\theta\\ &= \cos^4\theta-\cos^2\theta\\ &= \text{RHS}\end{aligned}$$

Question 7

$$\text{LHS} = \sin^2\theta\ \tan^2\theta$$

$$= \sin^2\theta . \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\left(1-\cos^2\theta\right).\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$

$$= \tan^2\theta - \sin^2\theta$$

$$= \text{RHS}$$

Question 8

$$\text{LHS} = \left(1+\sin\theta\right)\left(1-\sin\theta\right)$$

$$= 1-\sin^2\theta$$

$$= 1-\left(1-\cos^2\theta\right)$$

$$= 1-1+\cos^2\theta$$

$$= \cos^2\theta - 1 + 1$$

$$= \left(\cos\theta+1\right)\left(\cos\theta-1\right)+1$$

$$= \text{RHS}$$

Question 9

$$\text{LHS} = \sin\theta\tan\theta + \cos\theta$$

$$= \sin\theta\frac{\sin\theta}{\cos\theta} + \cos\theta$$

$$= \frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta}{1}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \text{RHS}$$

Question 10

$$\begin{aligned}\text{LHS} &= 1 \div \left(1+\tan^2\theta\right)\\ &= 1 \div \left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\\ &= 1 \div \left(\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\right)\\ &= 1 \div \frac{1}{\cos^2\theta}\\ &= \cos^2\theta\\ &= \text{RHS}\end{aligned}$$

Question 11

$$\begin{aligned}\text{LHS} &= \frac{\cos^2\theta+2\cos\theta+1}{\sin^2\theta}\\ &= \frac{\left(\cos\theta+1\right)^2}{\left(1-\cos^2\theta\right)}\\ &= \frac{\left(\cos\theta+1\right)^2}{\left(1-\cos\theta\right)\left(1+\cos\theta\right)}\\ &= \frac{\cos\theta+1}{1-\cos\theta}\\ &= \text{RHS}\end{aligned}$$

Question 12

$$\begin{aligned}
\text{LHS} &= \frac{\sin\theta}{1-\cos\theta} - \frac{\cos\theta}{\sin\theta} \\
&= \frac{\sin^2\theta - \cos\theta(1-\cos\theta)}{\sin\theta(1-\cos\theta)} \\
&= \frac{\sin^2\theta - \cos\theta + \cos^2\theta}{\sin\theta(1-\cos\theta)} \\
&= \frac{1-\cos\theta}{\sin\theta(1-\cos\theta)} \\
&= \frac{1}{\sin\theta} \\
&= \text{RHS}
\end{aligned}$$

Question 13

$$\begin{aligned}
\text{LHS} &= \frac{1-\sin\theta\cos\theta-\cos^2\theta}{\sin^2\theta+\sin\theta\cos\theta-1} \\
&= \frac{1-\cos^2\theta-\sin\theta\cos\theta}{\sin^2\theta-1\sin\theta\cos\theta} \\
&= \frac{\sin^2\theta-\sin\theta\cos\theta}{-\cos^2\theta+\sin\theta\cos\theta} \\
&= \frac{\sin\theta(\sin\theta-\cos\theta)}{\cos\theta(-\cos\theta+\sin\theta)} \\
&= \tan\theta \\
&= \text{RHS}
\end{aligned}$$

Exercise 9B

Question 1

$$\begin{aligned}\text{LHS} &= \sin(360+\theta)\\ &= \sin 360\cos\theta + \cos 360\sin\theta\\ &= 0+\sin\theta\\ &= \sin\theta\\ &= \text{RHS}\end{aligned}$$

Question 2

$$\text{LHS} = \cos(360+\theta)$$
$$= \cos 360\cos\theta - \sin 360\sin\theta$$
$$= \cos\theta - 0$$
$$= \cos\theta$$
$$= \text{RHS}$$

Question 3

$$\begin{aligned}\text{LHS} &= \sin(360-\theta)\\ &= \sin 360\cos\theta - \cos 360\sin\theta\\ &= 0-\sin\theta\\ &= -\sin\theta\\ &= \text{RHS}\end{aligned}$$

Question 4

$$\begin{aligned}\text{LHS} &= \cos(360-\theta)\\ &= \cos 360\cos\theta + \sin 360\sin\theta\\ &= \cos\theta\\ &= \text{RHS}\end{aligned}$$

Question 5

$$
\begin{aligned}
\text{LHS} &= \sin(A+B) - \sin(A-B) \\
&= \sin A\cos B + \cos A\sin B - \left(\sin A\cos B - \cos A\sin B\right) \\
&= \cos A\sin B + \cos A\sin B \\
&= 2\cos A\sin B \\
&= \text{RHS}
\end{aligned}
$$

Question 6

$$
\begin{aligned}
\text{LHS} &= \cos(A-B) + \cos(A+B) \\
&= \cos A\cos B + \sin A\sin B + \cos A\cos B - \sin A\sin B \\
&= 2\cos A\cos B \\
&= \text{RHS}
\end{aligned}
$$

Question 7

$$
\begin{aligned}
\text{LHS} &= 2\cos(x - \frac{\pi}{6}) \\
&= 2\left(\cos x\cos\frac{\pi}{6} + \sin x\sin\frac{\pi}{6}\right) \\
&= 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) \\
&= \sqrt{3}\cos x + \sin x \\
&= \text{RHS}
\end{aligned}
$$

Question 8

$$
\begin{aligned}
\text{LHS} &= \tan\left(\theta + \frac{\pi}{4}\right) \\
&= \frac{\tan\theta + \tan\frac{\pi}{4}}{1 - \tan\theta\tan\frac{\pi}{4}} \\
&= \frac{\tan\theta + 1}{1 - \tan\theta} \\
&= \text{RHS}
\end{aligned}
$$

Question 9

$$\begin{aligned}
\text{LHS} &= \frac{\cos(A+B)}{\cos(A-B)} \\
&= \frac{\cos A\cos B-\sin A\sin B}{\cos A\cos B+\sin A\sin B} \\
&= \frac{\cos A\cos B-\sin A\sin B}{\cos A\cos B} \div \frac{\cos A\cos B+\sin A\sin B}{\cos A\cos B} \\
&= \left(1-\frac{\sin A\sin B}{\cos A\cos B}\right) \div \left(1+\frac{\sin A\sin B}{\cos A\cos B}\right) \\
&= \left(1-\tan A\tan B\right) \div \left(1+\tan A\tan B\right) \\
&= \frac{1-\tan A\tan B}{1+\tan A\tan B} \\
&= \text{RHS}
\end{aligned}$$

Question 10

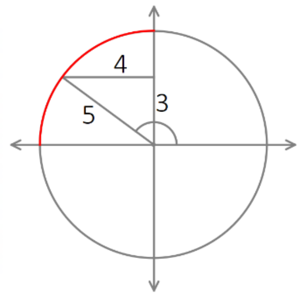
$$\begin{aligned}
\text{LHS} &= \sqrt{2}\left(\sin x-\cos x\right).\sin(x+45°) \\
&= \left(\sin x-\cos x\right)\sqrt{2}\left(\sin x\cos 45°+\cos x\cos 45°\right) \\
&= \left(\sin x-\cos x\right)\sqrt{2}\left(\sin x\frac{1}{\sqrt{2}}+\cos x\frac{1}{\sqrt{2}}\right) \\
&= (\sin x-\cos x)(\sin x+\cos x) \\
&= \sin^2 x-\cos^2 x \\
&= 1-\cos^2 x-\cos^2 x \\
&= 1-2\cos^2 x \\
&= \text{RHS}
\end{aligned}$$

Question 11

$$\begin{aligned}
\text{LHS} &= \tan(\theta + \frac{\pi}{4}) \\
&= \frac{\tan\theta + \tan\frac{\pi}{4}}{1 - \tan\theta\tan\frac{\pi}{4}} \\
&= \frac{\tan\theta + 1}{1 - \tan\theta} \\
&= \left(\frac{\sin\theta}{\cos\theta} + 1\right) \div \left(1 - \frac{\sin\theta}{\cos\theta}\right) \\
&= \left(\frac{\sin\theta + \cos\theta}{\cos\theta}\right) \div \left(\frac{\cos\theta - \sin\theta}{\cos\theta}\right) \\
&= \frac{(\sin\theta + \cos\theta)}{(\cos\theta - \sin\theta)} \times \frac{(\cos\theta + \sin\theta)}{(\cos\theta + \sin\theta)} \\
&= \frac{\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta + \cos\theta\sin\theta}{\cos^2\theta - \sin^2\theta} \\
&= \frac{1 + 2\sin\theta\cos\theta}{1 - \sin^2\theta - \sin^2\theta} \\
&= \frac{1 + 2\sin\theta\cos\theta}{1 - 2\sin^2\theta} \\
&= \text{RHS}
\end{aligned}$$

Question 1

a



$$\sin A = \frac{3}{5} \therefore \cos A = -\frac{4}{5}$$

$$\sin 2A = 2\sin A\cos A$$
$$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$$
$$= -\frac{24}{25}$$

b

$$\begin{aligned}\cos 2A &= 1 - 2\sin^2 A\\ &= 1 - 2\left(\frac{3}{5}\right)^2\\ &= 1 - \frac{18}{25}\\ &= \frac{7}{25}\end{aligned}$$

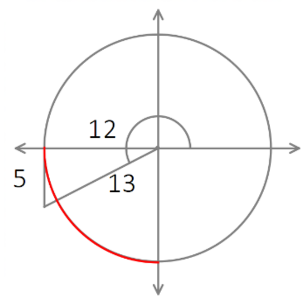
c

$$\begin{aligned}\tan 2A &= \frac{\sin 2A}{\cos 2A}\\ &= -\frac{24}{25} \div \frac{7}{25}\\ &= -3\frac{3}{7}\end{aligned}$$

Question 2

Given $\tan B = \frac{5}{12}$,

then $\sin B = -\frac{5}{13}$ and $\cos B = -\frac{12}{13}$



a

$$\begin{aligned}\sin 2B &= 2\sin B\cos B\\ &= 2\left(-\frac{5}{13}\right)\left(-\frac{12}{13}\right)\\ &= \frac{120}{169}\end{aligned}$$

b

$$\begin{aligned}\cos 2B &= 2\cos^2 B - 1\\ &= 2\left(\frac{-12}{13}\right)^2 - 1\\ &= \frac{119}{169}\end{aligned}$$

c

$$\begin{aligned}\tan 2B &= \frac{\sin 2B}{\cos 2B}\\ &= \frac{120}{169} \div \frac{119}{169}\\ &= \frac{120}{119}\end{aligned}$$

Question 3

a

$$6\sin A\cos A$$
$$=3\times 2\sin A\cos A \text{ bb}$$
$$=3\sin 2A$$

b

$$4\sin 2A\cos 2A$$
$$=2\times 2\sin 2A\cos 2A$$
$$=2\sin 4A$$

c

$$\sin\frac{A}{2}\cos\frac{A}{2}$$
$$=\frac{1}{2}\times 2\sin\frac{A}{2}\cos\frac{A}{2}$$
$$=\frac{1}{2}\sin A$$

Question 4

a

$$2\cos^2 2A-2\sin^2 2A$$
$$=2(\cos^2 2A-\sin^2 2A)$$
$$=2\cos 4A$$

b

$$1-2\sin^2\left(\frac{A}{2}\right)$$
$$=\cos A$$

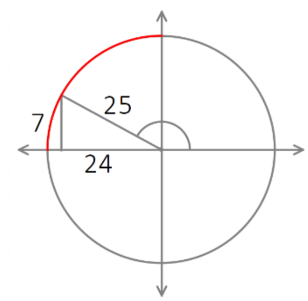
c

$$2\cos^2 2A-1$$
$$=\cos 4A$$

Question 5

Given $\cos\theta = \dfrac{-24}{25}$,

then $\sin\theta = \dfrac{7}{25}$ and $\tan\theta = -\dfrac{7}{24}$



a

$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta \\ &= 2\left(\frac{7}{25}\right)\left(-\frac{24}{25}\right) \\ &= -\frac{336}{625}\end{aligned}$$

b

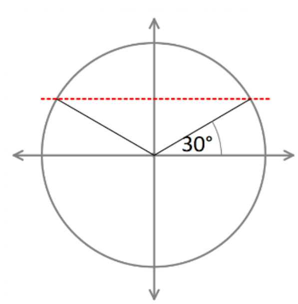
$$\begin{aligned}\cos 2\theta &= 2\cos^2\theta - 1 \\ &= 2.\left(\frac{24}{25}\right)^2 - 1 \\ &= \frac{527}{625}\end{aligned}$$

c

$$\begin{aligned}\tan 2\theta &= \frac{\sin 2\theta}{\cos 2\theta} \\ &= -\frac{336}{625} \div \frac{527}{625} \\ &= -\frac{336}{527}\end{aligned}$$

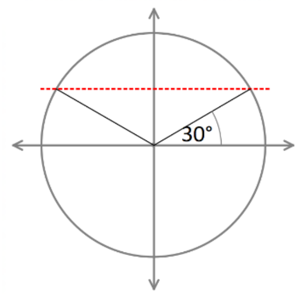
Question 6

$$\begin{aligned}4\sin x\cos x &= 1 \\ 2(2\sin 2x) &= 1 \\ 2\sin 2x &= 1 \\ \sin 2x &= \frac{1}{2} \\ 2x &= 30°, 150°, 390°, 510° \\ x &= 15°, 75°, 195°, 255°\end{aligned}$$



$0 \leq x \leq 360°$
$0 \leq 2x \leq 720°$

Question 7



$$\sin 2x + \cos x = 0$$
$$2\sin x\cos x + \cos x = 0$$
$$\cos x(2\sin x + 1) = 0$$
$$\cos x = 0 \text{ or } \sin x = -\frac{1}{2}$$
$$x = -90°, 90° \quad \text{or} \quad x = -150°, -30°$$
$$\therefore x = -150°, -90°, -30°, 90°$$

Question 8

$$2\sin 2x - \sin x = 0$$
$$2(2\sin x\cos x) - \sin x = 0$$
$$\sin x(4\cos x - 1) = 0$$
$$\sin x = 0 \text{ or } \cos x = \frac{1}{4}$$
$$x = 0°, 180°, 360° \text{ or } x = 75.5°, 284.5°$$
$$\therefore x = 0°, 75.5°, 180°, 284.5°, 360°$$

Question 9

$$2\sin x\cos x = \cos 2x \qquad 0 \le x \le 2\pi$$
$$\sin 2x = \cos 2x \qquad 0 \le 2x \le 4\pi$$
$$\frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\cos 2x}$$
$$\tan 2x = 1$$
$$2x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$
$$x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$$

Question 10

$$\begin{aligned} \cos 2x + 1 - \cos x &= 0 \\ 2\cos^2 x - 1 + 1 - \cos x &= 0 \\ \cos x(2\cos x - 1) &= 0 \\ \cos x = 0 \ \text{ or } \ \cos x &= \frac{1}{2} \\ x = \frac{\pi}{2}, \frac{3\pi}{2} \ \text{ or } \ x &= \frac{\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

$$\therefore x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$$

Question 11

$$\begin{aligned} \cos 2x + \sin x &= 0 \\ 1 - 2\sin^2 x + \sin x &= 0 \\ 2\sin^2 x - \sin x - 1 &= 0 \\ (2\sin x + 1)(\sin x - 1) &= 0 \\ \sin x = -\frac{1}{2} \ \text{ or } \ \sin x &= 1 \\ x = -\frac{5\pi}{6}, -\frac{\pi}{6} \ \text{ or } \ x &= \frac{\pi}{2} \end{aligned}$$

$$\therefore x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}$$

Question 12

$$\begin{aligned} 2\sin^2 x + 5\cos x + \cos 2x &= 3 \\ 2(1 - \cos^2 x) + 5\cos x + 2\cos^2 x - 1 &= 3 \\ 2 - 2\cos^2 x + 2\cos^2 x + 5\cos x - 1 &= 3 \\ 5\cos x &= 2 \\ \cos x &= 0.4 \end{aligned}$$

$$\therefore x = 66.4°, 293.6°, 426.4°$$

Question 13

$$\begin{aligned}\text{LHS} &= \sin 2\theta \tan\theta \\ &= 2\sin\theta\cos\theta \ \frac{\sin\theta}{\cos\theta} \\ &= 2\sin^2\theta \\ &= \text{RHS}\end{aligned}$$

Question 14

$$\begin{aligned}\text{LHS} &= \cos\theta \sin 2\theta \\ &= \cos\theta \ 2\sin\theta\cos\theta \\ &= 2\sin\theta\cos^2\theta \\ &= 2\sin\theta\left(1-2\sin^2\theta\right) \\ &= 2\sin\theta - 2\sin^3\theta \\ &= \text{RHS}\end{aligned}$$

Question 15

$$\begin{aligned}\text{LHS} &= \frac{1-\cos 2\theta}{1+\cos 2\theta} \\ &= \frac{1-\left(1-2\sin^2\theta\right)}{1+2\cos^2\theta-1} \\ &= \frac{2\sin^2\theta}{2\cos^2\theta} \\ &= \tan^2\theta \\ &= \text{RHS}\end{aligned}$$

Question 16

$$\begin{aligned}
\text{LHS} &= \sin\theta\,\tan\frac{\theta}{2}\\
&= 2\sin\frac{\theta}{2}\,\cos\frac{\theta}{2}\quad\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\\
&= 2\sin^2\frac{\theta}{2}\\
&= 2\left(1-\cos^2\frac{\theta}{2}\right)\\
&= 2-2\cos^2\frac{\theta}{2}\\
&= \text{RHS}
\end{aligned}$$

Question 17

$$\begin{aligned}
\text{LHS} &= \sin 4\theta\\
&= 2\times 2\sin 2\theta\cos 2\theta\\
&= 4s\sin\theta\cos\theta(\cos^2\theta-\sin^2\theta)\\
&= 4\sin\theta\cos^3\theta-4\sin^3\theta\cos\theta\\
&= \text{RHS}
\end{aligned}$$

Question 18

$$\begin{aligned}
\text{LHS} &= \frac{\sin 2\theta-\sin\theta}{1-\cos\theta+\cos 2\theta}\\
&= \frac{2\sin\theta\cos\theta-\sin\theta}{1-\cos\theta+2\cos^2\theta-1}\\
&= \frac{2\sin\theta\cos\theta-\sin\theta}{2\cos^2\theta-\cos\theta}\\
&= \frac{\sin\theta\left(2\cos\theta-1\right)}{\cos\theta\left(2\cos\theta-1\right)}\\
&= \tan\theta\\
&= \text{RHS}
\end{aligned}$$

Question 19

$$
\begin{aligned}
\text{LHS} &= \cos 4\theta \\
&= 2\cos^2 2\theta - 1 \\
&= 2\left(2\cos^2\theta - 1\right)^2 - 1 \\
&= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1 \\
&= 8\cos^4\theta - 8\cos^2\theta + 2 - 1 \\
&= 1 - 8\cos^2\theta + 8\cos^4\theta \\
&= \text{RHS}
\end{aligned}
$$

Exercise 9D

Question 1

$\sqrt{3^2+4^2}=5$

$5\left(\frac{3}{5}\cos\theta-\frac{4}{5}\sin\theta\right)=5(\cos\alpha\cos\theta-\sin\alpha\sin\theta)$

$\cos\alpha=\frac{3}{5},\sin\alpha=\frac{4}{5}\Rightarrow\alpha=53.1°$

$3\cos\theta-4\sin\theta=5\cos(\theta+53.1)°$

Question 2

$\sqrt{12^2+5^2}=13$

$13\left(\frac{12}{13}\cos\theta-\frac{5}{13}\sin\theta\right)=13(\cos\alpha\cos\theta-\sin\alpha\sin\theta)$

$\cos\alpha=\frac{12}{13},\sin\alpha=\frac{5}{13}\Rightarrow\alpha=22.6°$

$12\cos\theta-5\sin\theta=13\cos(\theta+22.6)°$

Question 3

$\sqrt{4^2+3^2}=5$

$5\left(\frac{4}{5}\cos\theta+\frac{3}{5}\sin\theta\right)=5(\cos\alpha\cos\theta+\sin\alpha\sin\theta)$

$\cos\alpha=\frac{4}{5},\sin\alpha=\frac{3}{5}\Rightarrow\alpha=0.64$

$4\cos\theta+3\sin\theta=5\cos(\theta-0.64)$

Question 4

$$\sqrt{7^2+24^2}=25$$

$$25\left(\frac{7}{25}\cos\theta+\frac{24}{25}\sin\theta\right)=25(\cos\alpha\cos\theta+\sin\alpha\sin\theta)$$

$$\cos\alpha=\frac{7}{25},\sin\alpha=\frac{24}{25}\Rightarrow\alpha=1.29$$

$$7\cos\theta+24\sin\theta=25\cos(\theta-1.29)$$

Question 5

$$\sqrt{5^2+12^2}=13$$

$$13(\frac{5}{13}\sin\theta+\frac{12}{13}\cos\theta)=13(\cos\alpha\sin\theta+\sin\alpha\cos\theta)$$

$$\cos\alpha=\frac{5}{13},\sin\alpha=\frac{12}{13}\Rightarrow\alpha=67.4°$$

$$5\sin\theta+12\cos\theta=13\sin(\theta+67.4)°$$

Question 6

$$\sqrt{7^2+24^2}=25$$

$$25(\frac{7}{25}\sin\theta+\frac{24}{25}\cos\theta)=25(\cos\alpha\sin\theta+\sin\alpha\cos\theta)$$

$$\cos\alpha=\frac{7}{25},\sin\alpha=\frac{24}{25}\Rightarrow\alpha=73.7°$$

$$7\sin\theta+24\cos\theta=25\sin(\theta+73.7)°$$

Question 7

$$\sqrt{4^2+3^2}=5$$

$$5(\frac{4}{5}\sin\theta-\frac{3}{5}\cos\theta)=25(\cos\alpha\sin\theta-\sin\alpha\cos\theta)$$

$$\cos\alpha=\frac{4}{5},\sin\alpha=\frac{3}{5}\Rightarrow\alpha=0.64$$

$$4\sin\theta-3\cos\theta=5\sin(\theta-0.64)$$

Question 8

$$\sqrt{2^2+3^2}=\sqrt{13}$$

$$\sqrt{13}\left(\frac{2}{\sqrt{13}}\sin\theta-\frac{3}{\sqrt{13}}\cos\theta\right)=\sqrt{13}(\cos\alpha\sin\theta-\sin\alpha\cos\theta)$$

$$\cos\alpha=\frac{2}{\sqrt{13}},\sin\alpha=\frac{3}{\sqrt{13}}\Rightarrow\alpha=0.98$$

$$2\sin\theta-3\cos\theta=\sqrt{13}\sin(\theta-0.98)$$

Question 9

$$4\sin\theta-3\cos\theta=5\sin(\theta-0.64)$$

$$2\sin\theta-3\cos\theta=\sqrt{13}\sin(\theta-0.98)$$

Question 10

a

$$R\cos(\theta-\alpha) = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$$

$$R = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\cos\theta + \sin\theta = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right)$$

$$\cos\alpha = \sin\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4}$$

$$\therefore \cos\theta + \sin\theta = \sqrt{2}\cos(\theta - \frac{\pi}{4})$$

b

Maximum value of $\cos(\theta-\frac{\pi}{4}) = 1$

$\therefore \sqrt{2}\cos(\theta-\frac{\pi}{4}) = \sqrt{2}$

Maximum occurs when $\cos(\theta-\frac{\pi}{4}) = 1$

$$(\theta-\frac{\pi}{4}) = 0$$

$$\theta = \frac{\pi}{4}$$

Question 11

$$R\cos(x-\alpha)=R\left(\cos x\cos\alpha+\sin x\sin\alpha\right)$$

$$R=\sqrt{3^2+4^2}=5$$

$$3\cos x+4\sin x=5(\frac{3}{5}\cos x+\frac{4}{5}\sin x)$$

$$\cos\alpha=\frac{3}{5},\sin\alpha=\frac{4}{5}\Rightarrow\alpha=0.93$$

$$3\cos x+4\sin x=5\cos(x-0.93)$$

$$5\cos(x-0.93)=2$$

$$\cos(x-0.93)=0.4$$

$$x-0.93=1.16,5.12$$

$$x=2.09,6.05$$

Question 12

$$R\cos(x-\alpha)=R\left(\cos x\cos\alpha+\sin x\sin\alpha\right)$$

$$R=\sqrt{10^2+5^2}=5\sqrt{5}$$

$$5\cos x+10\sin x=5\sqrt{5}\left(\frac{5}{5\sqrt{5}}\cos x+\frac{10}{5\sqrt{5}}\sin x\right)$$

$$\cos\alpha=\frac{5}{5\sqrt{5}},\sin\alpha=\frac{10}{5\sqrt{5}}\Rightarrow\alpha=1.107$$

$$\therefore 5\cos x+10\sin x=5\sqrt{5}\cos(x-1.107)$$

$$5\sqrt{5}\cos(x-1.107)=8$$

$$\cos(x-1.107)=\frac{8}{5\sqrt{5}}$$

$$x-1.107=\pm0.773$$

$$x=0.33,1.88$$

Question 13

$$R\cos(x-\alpha) = R\left(\cos x\cos\alpha + \sin x\sin\alpha\right)$$
$$R = \sqrt{2^2+5^2} = \sqrt{29}$$
$$5\cos x + 2\sin x = \sqrt{29}\left(\frac{5}{\sqrt{29}}\cos x + \frac{2}{\sqrt{29}}\sin x\right)$$
$$\cos\alpha = \frac{5}{\sqrt{29}}, \sin\alpha = \frac{2}{\sqrt{29}} \Rightarrow \alpha = 0.381$$
$$\therefore 5\cos x + 10\sin x = \sqrt{29}\cos(x-0.381)$$

$$\sqrt{29}\cos(x-0.381) = 3$$
$$\cos(x-0.381) = \frac{3}{\sqrt{29}}$$
$$x-0.381 = 0.980, 5.303$$
$$x = 1.36, 5.68$$

Exercise 9E

Question 1

$$\sec x = 2$$

$$\frac{1}{\cos x} = 2$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

Question 2

$$3\operatorname{cosec}^2 x = 4$$

$$\operatorname{cosec}^2 x = \frac{4}{3}$$

$$\frac{1}{\sin^2 x} = \frac{4}{3}$$

$$\sin^2 x = \frac{3}{4}$$

$$\sin x = \pm\frac{\sqrt{3}}{2}$$

$$x = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

Question 3

$$\sin x \sec x - 3\sin x = 0$$

$$\frac{\sin x}{\cos x} - 3\sin x = 0$$

$$\sin x\left(\frac{1}{\cos x} - 3\right) = 0$$

$$\sin x = 0 \quad \text{or} \quad \frac{1}{\cos x} - 3 = 0$$

$$x = 0°, 180°, 360° \quad \text{or} \quad \frac{1}{\cos x} = 3$$

$$\cos x = \frac{1}{3}$$

$$x = 70.5°, 289.5°$$

$$\therefore x = 0°, 70.5°, 180°, 289.5°, 360°$$

Question 4

$$\sec x(3 - \sec x) = \tan^2 x - 1$$

$$3\sec x - \sec^2 x = \sec^2 x - 1 - 1$$

$$2\sec^2 x - 3\sec x - 2 = 0$$

$$(2\sec x + 1)(\sec x - 2) = 0$$

$$2\sec x + 1 = 0 \quad \text{or} \quad \sec x - 2 = 0$$

$$\sec x = -\frac{1}{2} \qquad \sec x = 2$$

$$\frac{1}{\cos x} = -\frac{1}{2} \qquad \frac{1}{\cos x} = 2$$

$$\cos x = -2 \qquad \cos x = \frac{1}{2}$$

$$\text{no such } x \qquad x = \pm 60°$$

Question 5

$$5\cos x = \sec x$$

$$5\cos x = \frac{1}{\cos x}$$

$$\cos^2 x = \frac{1}{5}$$

$$\cos x = \pm\frac{1}{\sqrt{5}}$$

$$x = 63.4°, 116.6°, 243.4°, 296.6°$$

Question 6

$$\text{cosec}\left(x+\frac{\pi}{3}\right) = \sqrt{2}$$

$$\frac{1}{\sin\left(x+\frac{\pi}{3}\right)} = \sqrt{2}$$

$$\sin\left(x+\frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

$$x+\frac{\pi}{3} = \frac{3\pi}{4}, \frac{9\pi}{4}$$

$$x = \frac{5\pi}{12}, \frac{23\pi}{12}$$

Question 7

$$\sec^2 x + \sec x = 2$$
$$\sec^2 x + \sec x - 2 = 0$$
$$(\sec x + 2)(\sec x - 1) = 0$$
$$\sec x + 2 = 0 \text{ or } \sec x - 1 = 0$$
$$\sec x = -2 \qquad \sec x = 1$$
$$\frac{1}{\cos x} = -2 \qquad \frac{1}{\cos x} = 1$$
$$\cos x = -\frac{1}{2} \qquad \cos x = 1$$
$$x = 120°, 240° \qquad x = 0°, 360°$$
$$\therefore x = 0°, 120°, 240°, 360°$$

Question 8

$$2\cot^2 x + 5\operatorname{cosec} x - 1 = 0$$
$$2(\operatorname{cosec}^2 x - 1) + 5\operatorname{cosec} x - 1 = 0$$
$$2\operatorname{cosec}^2 x - 2 + 5\operatorname{cosec} x - 1 = 0$$
$$2\operatorname{cosec}^2 x + 5\operatorname{cosec} x - 3 = 0$$
$$(2\operatorname{cosec} x - 1)(\operatorname{cosec} x + 3) = 0$$
$$2\operatorname{cosec} x - 1 = 0 \text{ or } \operatorname{cosec} x + 3 = 0$$
$$\operatorname{cosec} x = \frac{1}{2} \qquad \operatorname{cosec} x = -3$$
$$\frac{1}{\sin x} = \frac{1}{2} \qquad \frac{1}{\sin x} = -3$$
$$\sin x = 2 \qquad \sin x = -\frac{1}{3}$$
$$\text{no such } x \qquad x = 3.48, 5.94$$

Question 9

$$\begin{aligned} \text{RHS} &= \sin^2\theta\,\cot^2\theta + \sin^2\theta \\ &= \sin^2\theta\,\frac{\cos^2\theta}{\sin^2\theta} + \sin^2\theta \\ &= \cos^2\theta + \sin^2\theta \\ &= 1 \\ &= \text{LHS} \end{aligned}$$

Question 10

$$\text{LHS} = \cot^2\theta(1-\cos^2\theta)$$

$$= \frac{\cos^2\theta}{\sin^2\theta}\sin^2\theta$$

$$= \cos^2\theta$$

$$= 1-\sin^2\theta$$

$$= \text{RHS}$$

Question 11

$$\begin{aligned} \text{LHS} &= 1+\cot^2\theta \\ &= \text{cosec}^2 x \\ &= \frac{1}{\sin^2 x}\,\frac{\cos^2 x}{\cos^2 x} \\ &= \frac{\cos^2 x}{\sin^2 x}\,\frac{1}{\cos^2 x} \\ &= \cot^2\theta\,\sec^2\theta \\ &= \text{RHS} \end{aligned}$$

Question 12

$$
\begin{aligned}
\text{LHS} &= (\sec\theta - 1)(\operatorname{cosec}\theta + \cot\theta) \\
&= \sec\theta\ \operatorname{cosec}\theta - \operatorname{cosec}\theta + \sec\theta\ \cot\theta - \cot\theta \\
&= \frac{1}{\cos\theta}\frac{1}{\sin\theta} - \frac{1}{\sin\theta} + \frac{1}{\cos\theta}\frac{\cos\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta} \\
&= \frac{1}{\cos\theta}\frac{1}{\sin\theta} - \frac{1}{\sin\theta} + \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} \\
&= \frac{1}{\cos\theta}\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} \\
&= \frac{1-\cos^2\theta}{\cos\theta\sin\theta} \\
&= \frac{\sin^2\theta}{\cos\theta\sin\theta} \\
&= \frac{\sin\theta}{\cos\theta} \\
&= \tan\theta \\
&= \text{RHS}
\end{aligned}
$$

Question 13

$$
\begin{aligned}
\text{LHS} &= \tan^4\theta - 1 \\
&= (\tan^2\theta - 1)(\tan^2\theta + 1) \\
&= (\tan^2\theta - 1)\sec^2\theta \\
&= \tan^2\theta\sec^2\theta - \sec^2\theta \\
&= \text{RHS}
\end{aligned}
$$

Question 14

$$
\begin{aligned}
\text{LHS} &= \frac{1+\sin\theta}{1-\sin\theta} \\
&= \frac{(1+\sin\theta)}{(1-\sin\theta)}\,\frac{(1+\sin\theta)}{(1+\sin\theta)} \\
&= \frac{\sin^2\theta+2\sin\theta+1}{1-\sin^2\theta} \\
&= \frac{\sin^2\theta+2\sin\theta+1}{\cos^2\theta} \\
&= \frac{\sin^2\theta}{\cos^2\theta}+\frac{2\sin\theta}{\cos^2\theta}+\frac{1}{\cos^2\theta} \\
&= \tan^2\theta+2\tan\theta.\frac{1}{\cos\theta}+\sec^2\theta \\
&= \tan^2\theta+2\tan\theta\sec\theta+\tan^2\theta+1 \\
&= 2\tan^2\theta+2\tan\theta\sec\theta+1 \\
&= \text{RHS}
\end{aligned}
$$

Question 15

$$
\begin{aligned}
\text{LHS} &= \frac{1+\sin\theta}{1-\sin\theta} \\
&= \frac{(1+\sin\theta)}{(1-\sin\theta)}\,\frac{(1+\sin\theta)}{(1+\sin\theta)} \\
&= \frac{1+2\sin\theta+\sin^2\theta}{1-\sin^2\theta} \\
&= \frac{1+2\sin\theta+\sin^2\theta}{\cos^2\theta} \\
&= \frac{\sin^2\theta}{\cos^2\theta}+\frac{2\sin\theta}{\cos^2\theta}+\frac{1}{\cos^2\theta} \\
&= \tan^2\theta+2\tan\theta\sec\theta+\sec^2\theta \\
&= \text{RHS}
\end{aligned}
$$

Question 16

$$\begin{aligned}
\text{LHS} &= \frac{1+\sec\theta}{1-\sec\theta} \\
&= \frac{(1+\sec\theta)}{(1-\sec\theta)}\frac{(1+\sec\theta)}{(1+\sec\theta)} \\
&= \frac{\sec^2\theta+2\sec\theta+1}{1-\sec^2\theta} \\
&= \frac{\sec^2\theta+2\sec\theta+1}{1-(\tan^2\theta+1)} \\
&= \frac{\sec^2\theta+2\sec\theta+1}{-\tan^2\theta} \\
&= -\frac{\sec^2\theta}{\tan^2\theta}-\frac{2\sec\theta}{\tan^2\theta}-\frac{1}{\tan^2\theta} \\
&= -\frac{1}{\cos^2\theta}\frac{\cos^2\theta}{\sin^2\theta}-\frac{2}{\cos\theta}\frac{\cos^2\theta}{\sin^2\theta}-\cot^2\theta \\
&= -\frac{1}{\sin^2\theta}-\frac{2}{\sin\theta}\frac{\cos\theta}{\sin\theta}-(\text{cosec}^2\theta-1) \\
&= -\text{cosec}^2\theta-2\text{cosec}\theta\cot\theta-\text{cosec}^2\theta+1 \\
&= 1-2\text{cosec}^2\theta-2\cot\theta\,\text{cosec}\theta \\
&= \text{RHS}
\end{aligned}$$

Exercise 9F

Question 1

$$\begin{aligned}\cos 3x\cos 2x &= \frac{1}{2}\left[\cos(3x+2x)+\cos(3x-2x)\right]\\ &= \frac{1}{2}\cos 5x+\frac{1}{2}\cos x\end{aligned}$$

Question 2

$$\begin{aligned}2\sin 3x\sin x &= \frac{1}{2}\left[\cos(3x-x)-\cos(3x+x)\right]\\ &= \frac{1}{2}\cos 2x-\frac{1}{2}\cos 4x\end{aligned}$$

Question 3

$$\begin{aligned}\sin 7x\cos x &= \frac{1}{2}\left[\sin(7x+x)+\sin(7x-x)\right]\\ &= \frac{1}{2}\sin 8x+\frac{1}{2}\sin 6x\end{aligned}$$

Question 4

$$\begin{aligned}\cos 3x\sin x &= \frac{1}{2}\left[\sin(3x+x)-\sin(3x-x)\right]\\ &= \frac{1}{2}\sin 4x-\frac{1}{2}\sin 2x\end{aligned}$$

Question 5

$$\begin{aligned}\cos 5x+\cos x &= 2\cos\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right)\\ &= 2\cos 3x\cos 2x\end{aligned}$$

Question 6

$$\begin{aligned}\cos 5x - \cos x &= -2\sin\left(\frac{5x+x}{2}\right)\sin\left(\frac{5x-x}{2}\right)\\ &= -2\sin 3x \sin 2x\end{aligned}$$

Question 7

$$\begin{aligned}\sin 6x + \sin 2x &= 2\sin\left(\frac{6x+2x}{2}\right)\cos\left(\frac{6x-2x}{2}\right)\\ &= 2\sin 4x \cos 2x\end{aligned}$$

Question 8

$$\begin{aligned}\sin 5x - \sin 3x &= 2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\\ &= 2\cos 4x \sin x\end{aligned}$$

Question 9

$$\begin{aligned}\sin 75^\circ \cos 15^\circ &= \frac{1}{2}\left(\sin 90^\circ + \sin 60^\circ\right)\\ &= \frac{1}{2}(1+\frac{\sqrt{3}}{2})\\ &= \frac{2+\sqrt{3}}{4}\end{aligned}$$

Question 10

$$\begin{aligned}\sin 75^\circ + \sin 15^\circ &= 2\sin 45^\circ \cos 30^\circ\\ &= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\\ &= \frac{\sqrt{6}}{2}\end{aligned}$$

Question 11

$$4.\frac{1}{2}\left(\sin 9x+\sin 5x\right)=\sqrt{3}+2\sin 9x \qquad (0^\circ \le x \le 180^\circ \to 0^\circ \le 5x \le 900^\circ)$$

$$2\sin 9x+2\sin 5x=\sqrt{3}+2\sin 9x$$

$$2\sin 5x=\sqrt{3}$$

$$\sin 5x=\frac{\sqrt{3}}{2}$$

$$5x=60^\circ,120^\circ,420^\circ,480^\circ,780^\circ,840^\circ$$

$$x=12^\circ,24^\circ,84^\circ,96^\circ,156^\circ,168^\circ$$

Question 12

$$\sin 7x+\sin 3x=\sin 5x \qquad (0\le x\le \pi \to 0\le 2x\le 2\pi \ \&\ 0\le 5x\le 5\pi)$$

$$2\sin 5x\cos 2x=\sin 5x$$

$$2\sin 5x\cos 2x-\sin 5x=0$$

$$\sin 5x(2\cos 2x-1)=0$$

$$\sin 5x=0 \text{ or } 2\cos 2x-1=0$$

$$5x=0,\pi,2\pi,3\pi,4\pi,5\pi \text{ or } \cos 2x=\frac{1}{2}$$

$$x=0,\frac{\pi}{5},\frac{2\pi}{5},\frac{3\pi}{5},\frac{4\pi}{5},\pi \qquad 2x=\frac{\pi}{3},\frac{5\pi}{3}$$

$$x=\frac{\pi}{6},\frac{5\pi}{6}$$

$$\therefore x=0,\frac{\pi}{6},\frac{\pi}{5},\frac{2\pi}{5},\frac{3\pi}{5},\frac{4\pi}{5},\frac{5\pi}{6},\pi$$

Question 13

$$\sin 3x-\sin x=0$$

$$2\cos 2x\sin x=0$$

$$\cos 2x=0 \text{ or } \sin x=0$$

$$2x=90^\circ,270^\circ,450^\circ,630^\circ \text{ or } x=0^\circ,180^\circ,360^\circ$$

$$x=45^\circ,135^\circ,225^\circ,315^\circ$$

$$\therefore x=0^\circ,45^\circ,135^\circ,180^\circ,225^\circ,315^\circ,360^\circ$$

Question 14

$$\sin 5x \cos 3x = \sin 6x \cos 2x$$

$$\frac{1}{2}[\sin 8x + \sin 2x] = \frac{1}{2}[\sin 8x + \sin 4x]$$

$$\sin 8x + \sin 2x = \sin 8x + \sin 4x$$

$$\sin 2x = \sin 4x$$

$$\sin 4x - \sin 2x = 0$$

$$2\cos 3x \sin x = 0$$

$$\sin x = 0 \text{ or } \cos 3x = 0$$

$$x = -\pi, 0, \pi, \text{ or } 3x = -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$x = -\frac{5\pi}{6}, -\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

$$\therefore x = -\pi, -\frac{5\pi}{6}, -\frac{\pi}{2}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi$$

Question 15

$$\text{LHS} = \frac{\sin A + \sin B}{\cos A + \cos B}$$

$$= \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$

$$= 2\tan\left(\frac{A+B}{2}\right)$$

$$= \text{RHS}$$

Question 16

$$\begin{aligned}\text{LHS} &= \sqrt{2}\cos(2x-\frac{\pi}{4})-\frac{\sin 7x+\sin 3x}{2\sin 5x}\\ &= \sqrt{2}\left(\cos 2x\cos\frac{\pi}{4}+\sin 2x\sin\frac{\pi}{4}\right)-\frac{2\sin 5x\cos 2x}{2\sin 5x}\\ &= \sqrt{2}\left(\frac{\sqrt{2}}{2}\cos 2x+\frac{\sqrt{2}}{2}\sin 2x\right)-\cos 2x\\ &= \cos 2x+\sin 2x-\cos 2x\\ &= \sin 2x\\ &= \text{RHS}\end{aligned}$$

Question 17

$$\begin{aligned}\text{LHS} &= \cos 8A\cos 2A-\cos 7A\cos 3A+\sin 5A\sin A\\ &= \frac{1}{2}(\cos 10A+\cos 6A)-\frac{1}{2}(\cos 10A+\cos 4A)+\frac{1}{2}(\cos 4A-\cos 6A)\\ &= \frac{1}{2}\cos 10A+\frac{1}{2}\cos 6A-\frac{1}{2}\cos 10A-\frac{1}{2}\cos 4A+\frac{1}{2}\cos 4A-\frac{1}{2}\cos 6A\\ &= 0\\ &= \text{RHS}\end{aligned}$$

Question 18

$$\begin{aligned}\text{LHS} &= 4\sin 3A\sin 2A\cos A\\ &= 4\sin 2A\left[\frac{1}{2}(\sin 4A+\sin 2A)\right]\\ &= 2\sin 2A\sin 4A+2\sin^2 2A\\ &= 2\left[\frac{1}{2}(\cos 2A-\cos 6A)\right]+2\left[\frac{1}{2}(\cos 0-\cos 4A)\right]\\ &= \cos 2A-\cos 6A+\cos 0-\cos 4A\\ &= 1+\cos 2A-\cos 4A-\cos 6A\\ &= \text{RHS}\end{aligned}$$

Exercise 9G

Question 1

$$\sin x = 0.5$$
$$x = 30°, 150°$$
$$x = \begin{cases} 30° + 360n° \\ 150° + 360n° \end{cases}, n \in \mathbb{Z}$$

Question 2

$$\cos x = 1$$
$$x = 0°$$
$$x = 360n°, n \in \mathbb{Z}$$

Question 3

$$\tan x = -\frac{1}{\sqrt{3}}$$
$$x = -30° \text{ (closest solution to } 0°\text{)}$$
$$x = -30° + 180n°, n \in \mathbb{Z}$$

Question 4

$$\sin(2x + 30°) = 1$$
$$2x + 30° = 90° + 360n°$$
$$2x = 60° + 360n°$$
$$x = 30° + 180n°, n \in \mathbb{Z}$$

Question 5

$$\begin{aligned}
\cos(3(x-20°)) &= 0.7 \\
3(x-20°) &= \pm 45.6° + 360n° \\
x-20° &= \pm 15.2° + 120n° \\
x &= \begin{cases} 4.8° + 120n° \\ 35.2° + 120n° \end{cases},\ n \in \mathbb{Z}
\end{aligned}$$

Question 6

$$\begin{aligned}
\tan(2(x+10°)) &= 0.8 \\
2(x+10°) &= 38.66° + 180n° \\
x+10° &= 19.3n° + 90n° \\
x &= 9.3° + 90n°, n \in \mathbb{Z}
\end{aligned}$$

Question 7

$$\begin{aligned}
4\sin x \cos x &= -1 \\
4\left[\frac{1}{2}\left(\sin 2x + \sin 0\right)\right] &= -1 \\
2(\sin 2x + 0) &= -1 \\
\sin 2x &= -\frac{1}{2} \\
2x &= \frac{7\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n \\
x &= \begin{cases} \dfrac{7\pi}{12} + \pi n \\ \dfrac{11\pi}{12} + \pi n \end{cases}, n \in \mathbb{Z}
\end{aligned}$$

Question 8

$$\sin^3 x + \sin x \cos^2 x = \cos x$$
$$\sin^3 x + \sin x \cos^2 x - \cos x = 0$$
$$\sin x(1 - \cos^2 x) + \sin x \cos^2 x - \cos x = 0$$
$$\sin x - \sin x \cos^2 x + \sin x \cos^2 x - \cos x = 0$$
$$\sin x - \cos x = 0$$
$$\sin x = \cos x$$
$$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$$
$$\tan x = 1$$
$$x = \frac{\pi}{4} + \pi n, n \in \mathbb{Z}$$

Question 9

$$\cos^2 x - \sin^2 x = 1$$
$$1 - \sin^2 x - \sin^2 x = 1$$
$$1 - 2\sin^2 x = 1$$
$$-2\sin^2 x = 0$$
$$\sin^2 x = 0$$
$$\sin x = 0$$
$$x = 0 + 2\pi n, \pi + 2\pi n,\ n \in \mathbb{Z}$$
$$= \pi n,\ n \in \mathbb{Z}$$

Question 10

$$\sin 2x\cos x+\cos 2x\sin x=0.5$$

$$\frac{1}{2}\left(\sin 3x+\sin x\right)+\frac{1}{2}\left(\sin 3x-\sin x\right)=0.5$$

$$\frac{1}{2}\sin 3x+\frac{1}{2}\sin x+\frac{1}{2}\sin 3x-\frac{1}{2}\sin x=0.5$$

$$\sin 3x=0.5$$

$$3x=\frac{\pi}{6}+2\pi n,\frac{5\pi}{6}+2\pi n,n\in\mathbb{Z}$$

$$x=\begin{cases}\dfrac{\pi}{18}+\dfrac{2\pi}{3}n\\ \dfrac{5\pi}{18}+\dfrac{2\pi}{3}n\end{cases},\ n\in\mathbb{Z}$$

Question 11

$$\cos(4(x-1)=0.8$$

$$4(x-1)=\pm0.64+2\pi n,n\in Z$$

$$x-1=\pm0.16+\frac{\pi}{2}n,n\in Z$$

$$x=\begin{cases}0.84+\dfrac{\pi}{2}n\\ 1.16+\dfrac{\pi}{2}n\end{cases},n\in Z$$

Question 12

$$2\sin 3x\sin x+\cos 4x=0.5$$

$$2\left[\frac{1}{2}\left(\cos 2x-\cos 4x\right)\right]+\cos 4x=0.5$$

$$\cos 2x-\cos 4x+\cos 4x=0.5$$

$$\cos 2x=0.5$$

$$2x=\pm\frac{\pi}{3}+2\pi n,\ n\in\mathbb{Z}$$

$$x=\pm\frac{\pi}{6}+\pi n,\ n\in\mathbb{Z}$$

Question 13

$$\cos(3x-\frac{\pi}{4})=0$$

$$3x-\frac{\pi}{4}=\pm\frac{\pi}{2}+2\pi n,\ n\in\mathbb{Z}$$

$$3x=-\frac{\pi}{4}+2\pi n,\ \frac{3\pi}{4}+2\pi n, n\in\mathbb{Z}$$

$$x=\begin{cases}-\frac{\pi}{12}+\frac{2\pi}{3}n \\ \frac{\pi}{4}+\frac{2\pi}{3}n\end{cases},\ n\in\mathbb{Z}$$

Question 14

$$\sin(\frac{\pi}{4}(3x-1))=0.25$$

$$\frac{\pi}{4}(3x-1)=0.25+2\pi n, 2.89+2\pi n,\ n\in\mathbb{Z}$$

$$3x-1=\frac{4}{\pi}\left(0.25+2\pi n\right),\frac{4}{\pi}\left(2.89+2\pi n\right),\ n\in\mathbb{Z}$$

$$=\frac{1}{\pi}+8n, 3.68+8n,\ \ n\in\mathbb{Z}$$

$$3x=1.32+8n, 4.68+8n,\ \ n\in\mathbb{Z}$$

$$x=\begin{cases}0.44+\frac{8n}{3} \\ 1.56+\frac{8n}{3}\end{cases},\ n\in\mathbb{Z}$$

Exercise 9H

Question 1

a Amplitude = $3 \therefore a = 3$

Period : $2\pi \therefore b = 1$

$y = 3\sin x$

b Amplitude = $4 \therefore a = 4$

Period : $2\pi \therefore b = 1$

$y = 4\sin x$

c Amplitude = $3 \therefore a = 3$

Period : $2\pi \therefore b = 1$

$y = -3\sin x$

d Amplitude = $4 \therefore a = 4$

Period : $2\pi \therefore b = 1$

$y = -4\sin x$

Question 2

a Amplitude = $3 \therefore a = 3$

Period : $\frac{2\pi}{b} = \pi \therefore b = 2$

$y = 3\sin 2x$

b Amplitude = $4 \therefore a = 4$

Period : $\frac{2\pi}{b} = 3\pi \therefore b = \frac{2}{3}$

$y = 4\sin\left(\frac{2}{3}x\right)$

c Amplitude = $4 \therefore a = 4$

Period : $\frac{2\pi}{b} = 5 \therefore b = \frac{2\pi}{5}$

$y = 4\sin\left(\frac{2\pi}{5}x\right)$

d Amplitude = $5 \therefore a = 5$

Period : $\frac{2\pi}{b} = 6 \therefore b = \frac{\pi}{3}$

$y = -5\sin\left(\frac{\pi}{3}x\right)$

Question 3

a Mean value $= 2 \therefore d = 2$

Amplitude $= 3 \therefore a = 3$

Period : $\frac{2\pi}{b} = 2\pi \therefore b = 1$

$y = 2 + 3\sin x$

b Mean value $= -2 \therefore d = -2$

Amplitude $= 4 \therefore a = 4$

Period : $\frac{2\pi}{b} = 2\pi \therefore b = 1$

$y = -2 - 4\sin x$

Question 4

a Amplitude $= 3 \therefore a = 3$

Period : $\frac{2\pi}{b} = 2\pi \therefore b = 1$

$\sin\frac{\pi}{2} = 0 \therefore c = -\frac{\pi}{2}$

$y = 3\sin\left(x - \frac{\pi}{2}\right)$

b Amplitude $= 4 \therefore a = 4$

Period : $\frac{2\pi}{b} = 2\pi \therefore b = 1$

$\sin(-\frac{\pi}{2}) = 0 \therefore c = \frac{\pi}{2}$

$y = 4\sin\left(x + \frac{\pi}{2}\right)$

or

Amplitude $= 4 \therefore a = 4$

Period : $\frac{2\pi}{b} = 2\pi \therefore b = 1$

$\sin\frac{\pi}{2} = 0 \therefore c = -\frac{\pi}{2}$

$y = -4\sin\left(x - \frac{\pi}{2}\right)$

Question 5

a Amplitude = 5 $\therefore a = 5$

Period : $\frac{2\pi}{b} = 8 \therefore b = \frac{\pi}{4}$

$\sin(2) = 0 \therefore c = -2$

$y = 5\sin\left(\frac{\pi}{4}(x-2)\right)$

b

Amplitude = 4 $\therefore a = 4$

Period : $\frac{2\pi}{b} = 10 \therefore b = \frac{\pi}{5}$

$\sin(3) = 0 \therefore c = -3$

$y = 4\sin\left(\frac{\pi}{5}(x-3)\right)$

Question 6

a Amplitude = $\left(\dfrac{10-4}{2}\right)$=3 $\therefore a = 3$

Period : $\frac{2\pi}{b} = 8 \therefore b = \frac{\pi}{4}$

Mean value $= \left(\dfrac{4+10}{2}\right) = 7 \therefore d = 7$

$\sin(1) = 7 \therefore c = -1$

$y = 3\sin\left(\dfrac{\pi}{4}(x-1)\right) + 7$

b Amplitude = $\left(\dfrac{9-5}{2}\right) = 2 \therefore a = 2$

Max value occurs when $x = 25$

Next min value occurs when $x = 55$

This difference of 30 represent half a cycle

Period : $\frac{2\pi}{b} = 60 \therefore b = \frac{\pi}{30}$

Mean value $= 7 \therefore d = 7$

$\sin(10) = 7 \therefore c = -10$

$y = 2\sin\left(\dfrac{\pi}{30}(x-10)\right) + 7$

Question 7

$a = \left(\frac{17-7}{2}\right) = 5$

$d = \left(\frac{17+7}{2}\right) = 12$

Period: $\frac{2\pi}{b} = 365 \therefore b = \frac{2\pi}{365}$

Graph starts at mean value $\therefore c = 0$

$h = 5\sin\left(\frac{2\pi}{365}t\right) + 12$

Question 8

a $a = \left(\frac{16-4}{2}\right) = 6$

$d = \left(\frac{16+4}{2}\right) = 10$

Period: $\frac{2\pi}{b} = 12.5 \therefore b = \frac{4\pi}{25}$

Graph starts at min value $\therefore c = 0$

$d = -6\cos\left(\frac{4\pi}{25}t\right) + 10$

b $a = \left(\frac{16-4}{2}\right) = 6$

$d = \left(\frac{16+4}{2}\right) = 10$

Period: $\frac{2\pi}{b} = 12.5 \therefore b = \frac{4\pi}{25}$

Graph has a mean value when $t = \frac{12.5}{4} \therefore c = -\frac{25}{8}$

$d = 6\sin\left(\frac{4\pi}{25}\left(t - \frac{25}{8}\right)\right) + 10$

Question 9

a $a = \left(\frac{9-3}{2}\right) = 3$

$d = \left(\frac{9+3}{2}\right) = 6$

Period: $\frac{2\pi}{b} = 6 \therefore b = \frac{\pi}{3}$

Graph has a max value when $t = 2 \therefore c = -2$

$h = 3\cos\left(\frac{\pi}{3}(t-2)\right) + 6$

b $a = \left(\frac{9-3}{2}\right) = 3$

$d = \left(\frac{9+3}{2}\right) = 6$

Period: $\frac{2\pi}{b} = 6 \therefore b = \frac{\pi}{3}$

Graph has a mean value 1.5 seconds before the first max

i.e. when $t = 0.5 \therefore c = -0.5$

$h = 3\sin\left(\frac{\pi}{3}(t-0.5)\right) + 6$

Question 1

$$\sqrt{2}\sin 5x = 1$$
$$\sin 5x = \frac{1}{\sqrt{2}}$$
$$5x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}$$
$$x = \frac{\pi}{20}, \frac{3\pi}{20}, \frac{9\pi}{20}, \frac{11\pi}{20}, \frac{17\pi}{20}, \frac{19\pi}{20}$$

Question 2

$$\begin{aligned}\cos 3\theta &= \cos(2\theta + \theta)\\ &= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta\\ &= \left(2\cos^2\theta - 1\right)\cos\theta - 2\sin\theta\cos\theta\sin\theta\\ &= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta\\ &= 2\cos^3\theta - \cos\theta - 2\cos\theta\left(1 - \cos^2\theta\right)\\ &= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta\\ &= 4\cos^3\theta - 3\cos\theta\end{aligned}$$

Question 3

$$\begin{aligned}2\sin x\cos x &= \sqrt{3} - 2\sqrt{3}\sin^2 x\\ \sin 2x &= \sqrt{3}\cos 2x\\ \frac{\sin 2x}{\cos 2x} &= \sqrt{3}\\ \tan 2x &= \sqrt{3}\\ 2x &= 60°, 240°, 420°, 600°\\ x &= 30°, 120°, 210°, 300°\end{aligned}$$

Question 4

$$
\begin{aligned}
\tan^2 x &= 3(\sec x - 1) \\
\sec^2 x - 1 &= 3\sec x - 3 \\
\sec^2 x - 3\sec x + 2 &= 0 \\
(\sec x - 1)(\sec x - 2) &= 0 \\
\sec x = 1 \ \text{ or } \ \sec x &= 2 \\
\frac{1}{\cos x} = 1 \ \text{ or } \ \frac{1}{\cos x} &= 2 \\
\cos x = 1 \qquad \cos x &= \frac{1}{2} \\
x = 0 \qquad\qquad x &= -\frac{\pi}{3}, \frac{\pi}{3} \\
x &= -\frac{\pi}{3}, 0, \frac{\pi}{3}
\end{aligned}
$$

Question 5

$$
\begin{aligned}
4\sin 3x \cos x &= \sqrt{3} + 2\sin 2x \\
4\left[\tfrac{1}{2}(\sin 4x + \sin 2x)\right] &= \sqrt{3} + 2\sin 2x \\
2\sin 4x + 2\sin 2x &= \sqrt{3} + 2\sin 2x \\
2\sin 4x &= \sqrt{3} \\
\sin 4x &= \frac{\sqrt{3}}{2} \\
4x &= \frac{\pi}{3}, \frac{2\pi}{3} + 2\pi n, \qquad n \in \mathbb{Z} \\
x &= \begin{cases} \dfrac{\pi}{12} + \dfrac{\pi n}{2} \\ \dfrac{\pi}{6} + \dfrac{\pi n}{2} \end{cases}, n \in \mathbb{Z}
\end{aligned}
$$

Question 6

a

$$R^2 = 7^2 + 10^2$$
$$R = \sqrt{149}$$

$$R\sin(\theta - \alpha) = R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$$
$$7\sin\theta - 10\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$$

$$\sqrt{149}\cos\alpha = 7 \qquad 10 = \sqrt{149}\sin\alpha$$
$$\cos\alpha = \frac{7}{\sqrt{149}} \qquad \sin\alpha = \frac{10}{\sqrt{149}}$$

$$\tan\alpha = \frac{10}{7}$$
$$\alpha = 0.96$$

$$\therefore 7\sin\theta - 10\cos\theta = \sqrt{149}\sin(\theta - 0.96)$$

b

$\sqrt{149}\sin(\theta - 0.96)$

Minimum value of $\sin(\theta - 0.96) = -1$

$\therefore$ minimum value of $\sqrt{149}\sin(\theta - 0.96) = -\sqrt{149}$

$$\sin(\theta - 0.96) = -1$$
$$\theta - 0.96 = \frac{3\pi}{2}$$
$$\theta = \frac{3\pi}{2} + 0.96$$
$$= 5.67$$

Question 7

a, **b**

$$a = \frac{27.2 - 17}{2} = 5.1$$
$$d = \frac{27.2 + 17}{2} = 22.1$$
$$\frac{2\pi}{b} = 12 \Rightarrow b = \frac{\pi}{6}$$
$$c = -10$$
$$T = -5.1\sin\frac{\pi}{6}(x - 10) + 22.1 = 5.1\sin\frac{\pi}{6}(x - 10) + 22.1$$